KYKLOS, Vol. 49 – 1996 – Fasc. 4, 509 - 540

Inefficient and Locally Stable Trade Equilibria Under Scale Economies: Comparative Advantage Revisited

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This paper explores and analyzes the inefficient trade equilibria that can be introduced by scale economies despite the workings of the market mechanism. It thereby seeks to enrich the standard analysis of efficiency of equilibrium, upon which so much of welfare theory has focused. Research in recent decades on trade in commodities whose production entails scale economies – goods that are often the focus of public concern and public policy – have revealed circumstances that are in marked contrast to those in the classical model. It has long been known that trade models with scale economies are characterized by multiple equilibria (see, e.g., Marshall, Matthews 1949–1950, Kemp 1964 and Ethier 1979). Indeed, as Krugman (1991, pp. 651–652) remarks,

> 'In the emerging literature on increasing returns and externalities, multiple equilibria are not a nuisance but a central part of the story'.

On this point the present authors have shown (1992, 1994) that multiple equilibria do not merely exist but are normally vast in number, increasing exponentially with the number of traded commodities. This paper demonstrates that these many equilibria display some surprising attributes, and describes some previously-unrecognized ways in which the market mechanism's performance can fall short of ideal where scale economies are present.

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We will show that in a world of scale economies (Section I) market forces cannot always drive the economy to an equilibrium that entails efficient use of the economy's resources or that follows the dictates of comparative advantage; (Section II) under scale economies *many equilibria can be inefficient* in the standard sense of the term; (Section III) In contrast, *equilibria can be efficient even if they violate the normal comparative advantage conditions.*

Usually, an equilibrium is either shown to be efficient or, if it does not meet this standard, it is simply said to be inefficient. In this paper we extend these notions by introducing a measure of *the degree* to which any particular equilibrium falls short of perfect efficiency. We then apply this new measure to provide suggestive results about the equilibria of some particular trade models. We also formulate a more-flexible definition of *comparative advantage*, one that is useful in understanding efficiency in this more complex world. We accompany that by another new concept: *local efficiency*, entailing a comparison of the efficiency of a particular equilibrium point with that of other points in the neighborhood. We think these concepts bring some order to the issue of efficiency in the presence of economies of scale.

I. EFFICIENT AND INEFFICIENT EQUILIBRIA AND COMPARATIVE ADVANTAGE

Although our general orientation is toward large problems with large numbers of equilibria, we will begin our analysis with some small examples. These simple geometric examples illustrate the results that offer the greatest contrast with the classical case: that equilibria are not always efficient, and that equilibria can be efficient even if they violate comparative advantage.

Two basic definitions. In this paper we will use the term *perfectly specialized assignment* to connote any division of the task of producing the world's outputs among producer countries in such a way that no commodity is produced in more than one country. An assignment will be said to satisfy the *classical comparative advantage* requirements if when Country J produces good I and Country J′ produces good I′ then the ratio of the average product of Country J labor (assumed in our model to be the only input) in the production of I to its average product in I′ is higher than the corresponding ratio in Country J′. We leave for later discussion the appropriate *range* of input quantity over which the average products should be measured. This issue of range will in fact be critical for the results of Section III.

In Gomory and Baumol (1994) we proved that in a theoretical world of universal scale economies each and every perfectly specialized assignment

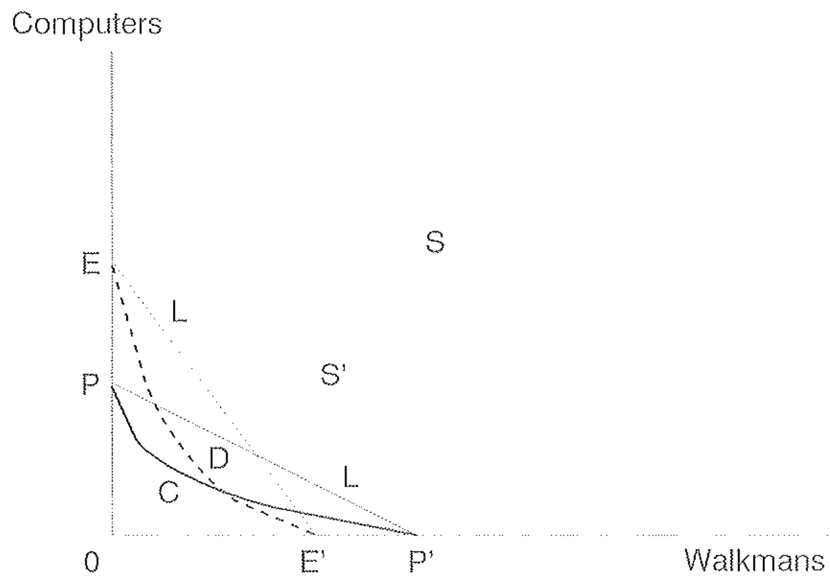
satisfies the requirements of equilibrium, and that, in addition, every such equilibrium is (locally) stable[1]. Moreover, the number of such specialized assignments is equal to the number of possible country-commodity combinations, and so it grows very rapidly when the number of goods or the number of countries in the model increases[2].

We will start our discussion with a small model which shows some of the properties of efficiency in the economies of scale case. The discussion is framed in terms of Ricardo's two countries, England and Portugal, but substitutes for his cloth and wine two more 'high-tech' products, computers and Walkman radios, in whose production we may expect scale economies. In such a two-good, two-country model it must be remembered that there are always exactly two perfectly specialized assignments. Portugal can produce all the Walkmans and England all the computers, or the reverse can be true.

1. It is an equilibrium at suitable prices and outputs because it then satisfies all the usual equilibrium requirements such as market clearing. The intuitive reason for the assignment's local stability is straightforward. Where there are scale economies, if Country J happens to be the exclusive producer of good I that will give it a cost advantage which Country J′ will be unable to overcome if it wants to embark on production of I, unless J′ launches into this activity on a large scale. The same will obviously be true if production of good I happens to be assigned exclusively to *any* other country. Thus any small (local) deviation from such a specialized assignment, implying that any attempted entry on a *small* scale into an industry by a country that is not currently supplying any of that industry's product, is foredoomed to failure. This means, of course, that the initial assignment is *locally* stable.
2. It is easy to show that with two countries and n commodities the number of perfectly specialized equilibria will be on the order of 2^n. This follows immediately from the argument of the previous footnote indicating why under substantial scale economies *every* perfectly specialized assignment of the production of the n goods between the two countries is a stable equilibrium. For labelling the goods 1,2,...,n, it is obvious that there are two choices of the country in which good 1 will be produced, and for each such assignment there are two possible assignments of good 2, etc.

Figure 1

One Specialized Equilibrium Dominates the Other



In *Figure 1*, PCP′ and EDE′ are the production frontiers for Portugal and England. Here, the convexity (downward) of the two frontiers represents the presence of scale economies, because towards the center of the frontier, where the country's production is unspecialized and the output of each good is relatively small, the output vector is held down (the output point is closer to the origin than it would be in the linear case)[3]. The two specialized solutions are S′ = (E′,P) and S = (P′,E). In the first of these England produces all the world's Walkmans and Portugal produces all of the computers, while in the second of these the production assignment is reversed. Both of these solutions are obviously locally stable equilibria at prices that clear the markets because if either country tries to produce a small quantity of the other's product it will fail because of its high costs. Yet, as shown in *Figure 1*, where the two frontiers intersect the one specialized point S clearly dominates the other, S′, so that the latter must be inefficient. This must always be so where the two countries'

3. In an unpublished paper Avinash Dixit has shown that convexity is sufficient but not necessary for scale economies.

production frontiers have an odd number of intersections, because then one country must be able to produce more Walkmans than the other and the other country must then be able to produce more computers than the first. Hence, the specialized equilibrium in which the first country produces all the Walkmans and the second country produces all the computers must then dominate the equilibrium in which the assignment of commodity production to the two countries is reversed. This example clearly shows that:

Theorem 1.1

In a world of scale economies an equilibrium can be inefficient.

Figure 2, in which the production frontiers of the two countries do not intersect, S and S′ remain the specialized solutions, but neither dominates the other. As a result, the efficiency of either of them is not immediately obvious. However, it is also possible to get some positive results, even for two-good problems. For example:

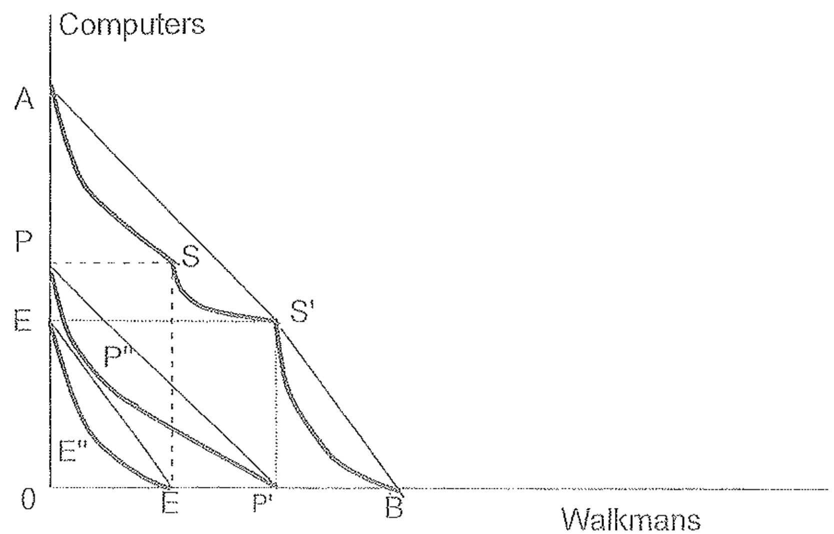
Theorem 1.2

One of the two production plans is always efficient. If the slope of chord EE′ is greater it is S and if the slope of chord PP′ is greater it is S′.

Proof: In *Figure 2* we have drawn the production frontier that would result if the chords were the actual production frontiers of the two countries, that is, if one were to make the problem linear in this way. It is, of course, well known that in such a two-by-two linear model one of the two specialized solutions will lie on the world production frontier, AS′B, and that point will satisfy the requirements of comparative advantage. In this case chord PP′ has the greater slope, meaning that Portugal's relative average product is greater in Walkman manufacture than in computer manufacture. For with a given supply of labor in each country (L_e and L_p, respectively), the slopes of the chords, EE′ and PP′ of the two countries are clearly $[(E/L_e) / (E'/L_e)]$ and $[(P/L_p) / (P'/L_p)]$, the ratios of the average products of labor in computer and Walkman production in the two countries. These ratios give us the traditional measure of comparative advantage. Thus, in a specialized equilibrium that satisfies comparative advantage Portugal will be the exclusive producer of Walkmans, as it is at S′. The point S′ is clearly efficient in this revised problem. It follows that it is *a fortiori* efficient in the original problem since, as the geometry clearly indicates, the production frontier of the original problem always lies at least as close to the origin as the production frontier of the problem based on the chords, so that any point of the original problem that lies on the linearized world frontier AS S′ B must obviously lie on the world frontier of the original problem. If the slopes were reversed we would apply the same reasoning and obtain the efficiency of the other specialized equilibrium S instead.

Figure 2

Efficient Equilibria and Comparative Advantage



Having just shown that in small-scale models of the sort under discussion an equilibrium that satisfies comparative advantage is necessarily efficient, we can use similar reasoning to show that the converse relationship does not hold – an equilibrium can be efficient even if it violates comparative advantage. In *Figure 2*, where one of the frontiers, PP′, lies entirely above the other, EE′, neither equilibrium point dominates the other. We have just seen that in the linearized model derived from the chords of the individual country frontiers solution point

S′ is efficient because it lies on the world (linearized) production frontier while equilibrium point S is not, because in the linear model S lies below the world frontier. However, because as a result of their (downward) convexity in the original problem the individual country frontiers lie below their chords almost throughout, the world frontier, ASS′B does *not* lie above S (it should be obvious that numerical examples of this sort are easy to construct). So both S and S′ really are efficient in the case shown in the graph. Since the production pattern in each of the two equilibria is the reverse of the other, the production patterns of only one of the two equilibria (S′) can be in accord with the traditional criterion of comparative advantage, so the other (S) has a production pattern that violates comparative advantage (compare the slopes of EE′ and PP′), yet is efficient. This example shows:

Theorem 1.3

Under scale economies an equilibrium can be efficient even if it violates the traditional comparative advantage requirement.

The preceding discussion also suggests that there is a general tradeoff between (a) the magnitude of the loss incurred as a result of violation of comparative advantage in an efficient specialized equilibrium and (b) the strength of the scale economies needed to render that equilibrium efficient. In terms of our diagrams, the greater the difference between the slopes of the chords of the production frontiers of the two countries, the greater the degree of convexity of those production frontiers that is necessary for an equilibrium that violates comparative advantage to be efficient. This is easily demonstrated geometrically in the two-country two-good case and we conjecture that an effect of this sort is true generally. The intuitive reason for this relationship indicates how scale economies weaken the influence of comparative advantage. For if a country specializes in the production of a good in which it has a comparative *dis*advantage the resulting loss of efficiency can by made up for as a result of its large output of that good if and only if the economies of scale are sufficiently strong.

Inefficient Equilibria: Can the Free Market Eliminate Them? But can't market forces be counted upon to destroy the inefficient equilibria that have just been shown to exist in a world of scale economies? This, surely, is what economists have long been taught about perfectly competitive models with universally diminishing returns. Is there not something similar to be expected here? After all, any inefficient equilibrium is necessarily an unrealized opportunity for mutual gain. Should not arbitrageurs or other businesspersons be counted upon to recognize such an opportunity and find ways to take advantage of it?

There are two fundamental reasons why this is not true in a world of scale economies. First, there is the fact that market signals are all local – indicating such things as marginal costs and marginal revenues that correspond to the partial derivatives of the profit function. They tell us in what direction to move in order to go upward on the profit hill on which the decision maker is currently located. But where there are millions of equilibria, each of them the peak of efficiency in its own neighborhood (see Section IV, below) going uphill from an initial position fortuitously selected by history may merely lead us toward the highest point on a nearby little hill. It can easily lead us away from the true global maximum, the top of a much-higher hill that may be far away.

Second, practical reality gives us another key reason showing why the unaided market will not generally be up to the task. Moving from one perfectly specialized assignment to another requires firms and countries to embark on the production of goods they are not currently turning out, and about whose production they have little knowledge or experience. Moreover, scale economies mean that to have any chance of success in such an endeavor, one must enter on a large scale. This requires the investors to undertake a very great risk, betting on what constitutes a leap into the unknown. Anyone with much business experience will confirm that such opportunities are hardly always recognized, and when recognized they are hardly always pursued. In sum, there is no reason to expect the market mechanism to take automatic advantage of the unrealized opportunities provided by inefficient equilibria. Under scale economies such equilibria can exist and can be locally stable.

Having completed these preliminaries, let us proceed toward the construction of a more-general theory of economic efficiency in the presence of scale economies.

II. ORIENTATION AND THE BASIC MODEL

Orientation. Our orientation in this paper is toward large problems with very large numbers of equilibria. Although we discuss both specialized and non-specialized equilibria, our analysis will emphasize specialized equilibria. There is considerable plausibility to this emphasis since it has been known for over three decades that, from a purely theoretical point of view, specialized equilibria are likely to play a dominant role in models with economies of scale because of the instability of most non-specialized equilibria. Furthermore (Gomory 1994)

showed that in large problems specialized equilibria play the dominant role in shaping the region of equilibria and determining the location of all equilibria, specialized and non-specialized alike. In this paper we will see once again the important role of specialized equilibria since we will show in Section IV that it is only the specialized and almost-specialized equilibria that are likely to be efficient in the usual sense.

However, in addition to specialized equilibria, there is one important class of non-specialized equilibria with properties that complement those of the specialized equilibria. These are equilibria with non-specialized (i.e., shared) production in some industries. These industries have production functions, that, after an initial range of rapidly decreasing average cost with increasing scale of production, eventually reach a minimum average cost, and thereafter become linear (the case of 'flat-bottomed average cost'). The instability that characterizes most shared equilibria is weak or non-existent for this type of industry. Equilibria that have shared production in some of these industries and specialized production in all the rest we will call *extended-specialized equilibria*.

The Basic Model. Our basic model consists of two countries (or two facilities) which we call Country 1 and Country 2. We will have production functions $f_{i,j}(l)$ that use the single input l to produce good i in Country j. If an amount of labor $l_{i,j}$ is used to produce the ith good in Country j, whose labor supply is L_j, the $l_{i,j}$ must satisfy the labor-availability inequalities

$$\sum_i l_{i,1} \leq L_1 \quad and \quad \sum_i l_{i,2} \leq L_2 . \tag{2.1}$$

Although we will sometimes refer to equilibria in our discussion or in our examples, the only property of an economic equilibrium that we will use is that its labor quantities satisfy (2.1)[4].

We define a *feasible production plan* $P = \{l_{i,j}\}$ for the quantities Q_i as a set of $l_{i,j}$ that satisfy (2.1) and has $f_{i,1}(l_{i,1}) + f_{i,2}(l_{i,2}) = Q_i$. We will say that the quantities Q_i are efficient if there is no set of $l_{i,j}$ that satisfy (2.1) and make more than the given output quantities, Q_i. We will assume that we have a feasible production plan P, possibly coming from an economic equilibrium. We would like to know if its resulting outputs, the Q_i, are an efficient set of goods.

There is one case that is completely straightforward. If P does not have equality in both the inequalities of (2.1), the unused labor can be used to make

4. For a full characterization of an equilibrium and proof that in our model *any* perfectly specialized assignment is a locally stable equilibrium, see Gomory and Baumol (1994).

additional goods. The total output would then strictly dominate the Q_i and they would not be efficient. In the remainder of this paper we will assume that the production pattern P that produces the Q_i does use up all the labor available in both countries.

We will say that the Q_i are efficient, or equivalently that P is efficient, if there is no feasible production plan that uses *strictly* less than the total labor of both countries[5]. In principle we could test for efficiency by solving a minimization problem: minimize the labor required subject to the non-linear conditions $f_{i,1}(l_{i,1}) + f_{i,2}(l_{i,2}) = Q_i$ for all i, and to (2.1), and see if that minimal amount is L_1+L_2. However, because of the non-linearities, this direct approach seems difficult and we will work instead with a linearized model.

III. THE LINEARIZED MODEL

This section of the paper deals with the classical efficiency concept and shows that in the case of scale economies comparative advantage, *defined in a somewhat different way*, is sufficient (but not necessary) to guarantee the efficiency of an equilibrium. We will also give a strong necessary condition.

Specialized Production. For a given i, one way to produce Q_i is to have it made solely in Country j. Then $Q_i = f_{i,j}(l'_{i,j})$ where $f_{i,j}$ is the production function for good i in Country j, and $l'_{i,j}$ is the amount of labor required in Country j to produce Q_i when Country j is the sole producer. If $Q_i > f_{i,j}(L_j)$ so that Q_i cannot be produced in Country j, even by all that country's labor, $l'_{i,j}$ is undefined. If $l'_{i,j}$ is defined we will say that $(i,j) \in D$, and if it is not defined we will say that $(i,j) \in D'$[6].

We will proceed by introducing related problems that replace the $f_{i,j}$ with linear production functions. These linearized production functions are related to, but are not the same as, those used in the proof of Theorem 1.1. We will refer to the problems obtained in this way as the *linearized problems*.

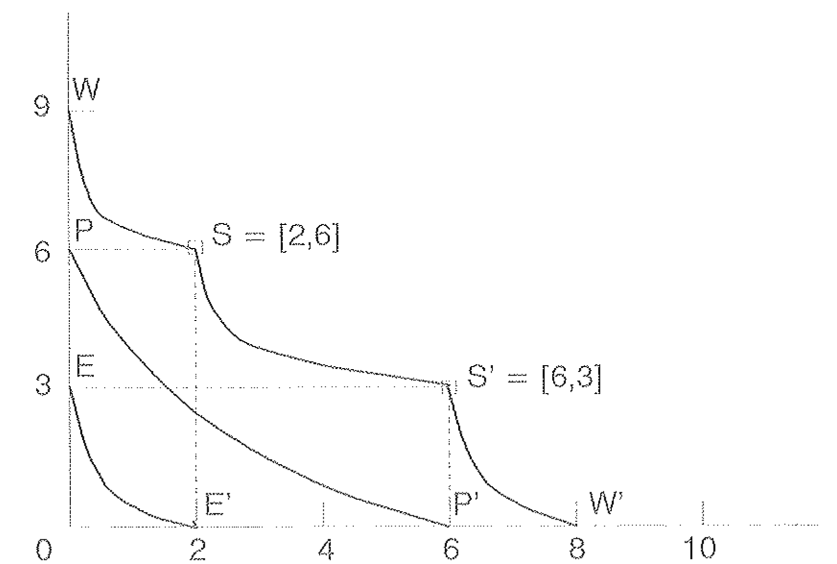
Efficiency in the linearized problems will turn out to be closely related to efficiency in the original non-linear problem. The linearized problems are both theoretically and computationally more tractable, and will be the natural basis for the concept of quantified efficiency (λ-efficiency) that we introduce in Section V.

5. One can, instead, use a dual approach that seeks $l_{i,j}$ employing all the labor of both countries and produces more than the Q_i.
6. While $(i,j) \in D'$ play a prominent role in very small problems they play a very small role in large problems where no single good uses up a large percentage of the labor force.

In defining a linearized problem the critical question is how to linearize. A natural answer is to take as the linearized production function the linear functions that make the Q_i with the same amount of labor as the $f_{i,j}$ themselves require. This is what we will do whenever possible. The exceptional cases, when the Q_i can't be produced even with the entire labor force of the country, we will handle slightly differently.

Figure 3

Production frontiers for Linearization



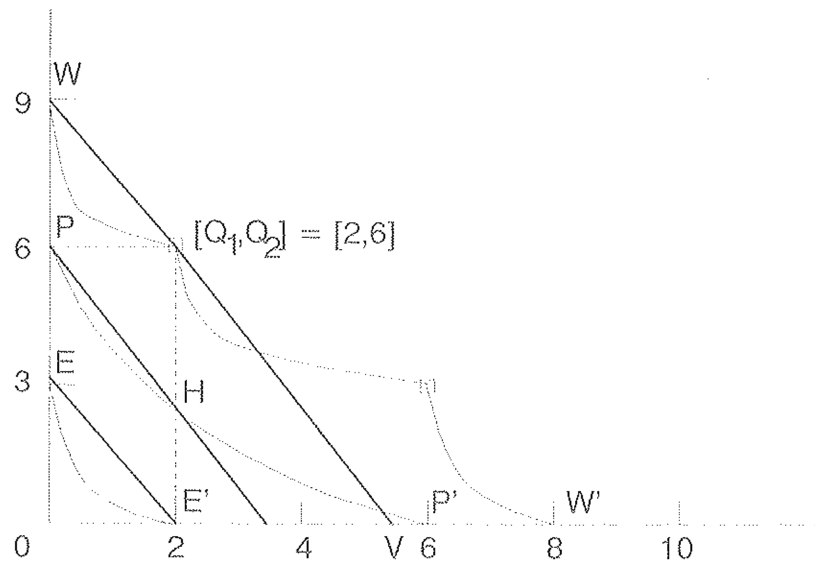
The Linearized Problem. We first define the *average productivities* $c_{i,j}$ by $c_{i,j} = f_{i,j}(l'_{i,j}) / l'_{i,j} = Q_i / l'_{i,j}$ for $(i,j) \in D$ and $c_{i,j} = f_{i,j}(L_j) / L_j$ for $(i,j) \in D'$. We then define the new linear production functions $L_{i,j}(l)$ by $L_{i,j}(l) = c_{i,j}l$. These linearized production functions clearly *depend on the choice of the Q*. The *linearized problem associated with the output quantities Q_i* is obtained by replacing the production functions $f_{i,j}(l)$ with the $L_{i,j}(l)$ defined from those Q_i. In *Figure 3* we see the original production frontiers for a typical economies of scale problem. Here, WSS' W' is the world frontier, obtained from the individual country frontiers EE' and PP'. In *Figure 4* the linearized production functions associated with the quantities $(Q_1,Q_2) = (2,6)$ are shown as the dark lines. The individual country linearized production frontiers are obtained by putting a straight line through the two intersections of the original country production frontier with the rectangle whose vertices are (0,0) and $(Q_1,Q_2) = (2,6)$, because at those intersection points the original production frontiers represent the use of a quantity of labor in the production of the pertinent good lc sufficient to produce the given output, Q_i of good *lc*. In *Figure 5* we see the linear problem associated with the quantities $(Q_1,Q_2) = (6,3)$. Clearly, when we use this construct different linear problems are associated with different quantities Q_i.

We will say that *the Q_i are L-efficient* if they are efficient in the associated linearized problem.

Next we define quantities $l^*_{i,j}$ that are analogous to the $l'_{i,j}$ of the original problem. These $l^*_{i,j}$ are the amounts of labor required to make the Q_i using the linearized production functions so $L_{i,j}(l^*_{i,j}) = Q_i$. For $(i,j) \in D$, $L_{i,j}(l'_{i,j}) = c_{i,j}l'_{i,j} = (Q_i / l'_{i,j})l'_{i,j} = Q_i$, so $l^*_{i,j} = l'_{i,j}$. For $(i,j) \in D'$, $L(L_j) = c_{i,j}\ L_j = (f_{i,j}(L_j) / L_j)L_j = f_{i,j}(L_j) < Q_i$, so $l^*_{i,j} > L_j$.

Figure 4

Linearization Based on Assignment S = [2,6]



Since l' and l^* are used repeatedly throughout the following discussion, it is useful to summarize their meaning. $l'_{i,j}$ is simply the quantity of labor needed to produce the given quantity of good i if it is done in country j alone. $l^*_{i,j}$ is the same, except where the given output of i is so large that it requires more than country j's entire labor force to produce it. In that case, $l^*_{i,j}$ is the quantity of labor that would be necessary to produce the given quantity of i, using a linearized production function based on the average productivity obtained when using the entire labor force of the country to produce good i.

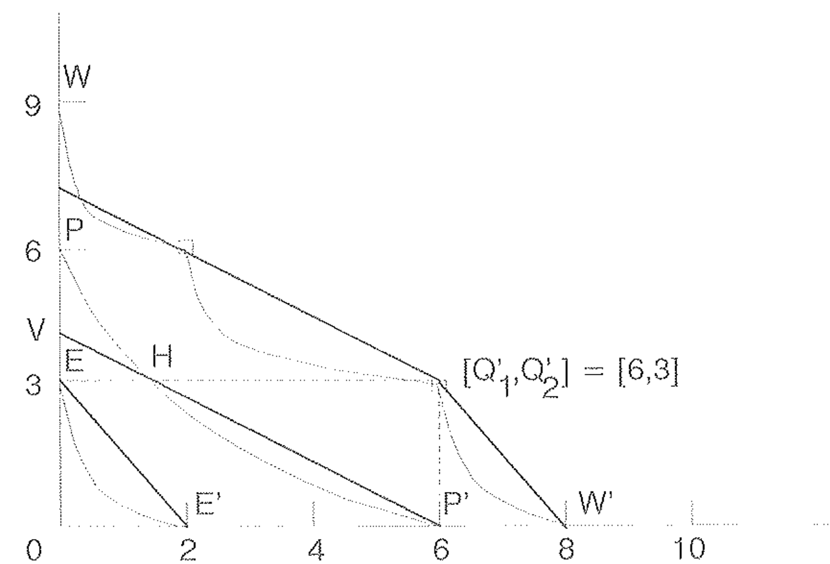
In contrast with the original problem, we can now easily characterize the labor inputs that will produce the Q_i using the linearized production functions. Consider any $y_{i,j}$ that satisfy:

$$0 \le y_{i,j} \le 1 \qquad y_{i,1} + y_{i,2} = 1,$$

$$\sum_i y_{i,1}\, l^*_{i,1} \le L_1 \quad and \quad \sum_i y_{i,2}\, l^*_{i,2} \le L_2. \tag{3.1}$$

Figure 5

Linearization Based on Assignment S′ = [6,3]



Each labor quantity $y_{i,j}l^*_{i,j}$ will, using the linearized production functions $L_{i,j}(l)$, produce exactly $L(y_{i,j}l^*_{i,j}) = c_{i,j}y_{i,j}l^*_{i,j} = y_{i,j}Q_i$. Therefore $y_{i,1}l^*_{i,1}$ and $y_{i,2}l^*_{i,2}$ together will produce $(y_{i,1} + y_{i,2})Q_i = Q_i$. Also the second line of (3.1) shows that these $y_{i,j}l^*_{i,j}$ will not overuse the quantities of labor available in the two countries. So the $y_{i,j}$ that satisfy (3.1) give the labor inputs $y_{i,j}l^*_{i,j}$ that produce the Q_i using the $L_{i,j}(l)$. We will often say that the $y_{i,j}$ produce the Q_i meaning that the labor quantities $y_{i,j}l^*_{i,j}$ do.

We can check for L-efficiency by solving the linear program that minimizes the total labor used in both countries to make the Q_i:

$$v = Min_y\, v(y) \quad v(y) = \sum_i (y_{i,1}\, l^*_{i,1} + y_{i,2}\, l^*_{i,2}) \ \ subject\ to$$

$$0 \le y_{i,j} \le 1 \qquad y_{i,1} + y_{i,2} = 1.$$

$$\sum_i y_{i,1}\, l^*_{i,1} \le L_1 \quad and \quad \sum_i y_{i,2}\, l^*_{i,2} \le L_2. \tag{3.2}$$

From this we can see whether or not the minimizing y requires the entire labor of both countries. *Clearly it is both a necessary and sufficient condition for L-efficiency that the minimizing y require all the labor of both countries.*

From standard linear programming we know[7] that a minimizing y will have $y_{i,j}$ equal to 0 or 1 for all but at most two i. Especially for large problems then, this minimizing y it is not very far from being a specialized production pattern. We will use this fact later. Also from standard linear programming we can write down the condition for y to minimize (3.2). The standard Kuhn-Tucker conditions applied to this specially structured program give: (proof available from the authors).

Lemma 3.1

y is a minimizing solution to (3.2) iff whenever the $y_{i,1}$ and $y_{k,2}$ from (3.2) are both positive we have $l^*_{i,1} / l^*_{i,2} \le l^*_{k,1} / l^*_{k,2}$.

This leads directly to:

7. This must be so because if we use $y_{i,1} + y_{i,2} = 1$ to eliminate the $y_{i,2}$ we are left with n $y_{i,1}$ variables and n + 2 slack variables, call them S_i and T_j. We cannot have $y_{i,1} = 0$ and $S_i = 0$ for the same i since $S_i = 0$ means $y_{i,1} = 1$. Thus, in a basic solution, in which n variables = 0, we can have at most two cases where $y_{i,1} > 0$ and $S_i > 0$. Later we will see that there is at most one $y_{i,1}$ that is neither 1 nor 0.

Theorem 3.2: Necessary and Sufficient Condition for L-Efficiency

The necessary and sufficient conditions for $y_{i,j}$ to be L-efficient is that (1) the $y_{i,j}l^*_{i,j}$ use all the labor of both countries and (2) they entail ratios $l^*_{i,1} / l^*_{i,2}$ that satisfy $l^*_{i,1} / l^*_{i,2} \le l^*_{k,1} / l^*_{k,2}$ when ever $y_{i,1} > 0$ and $y_{k,2} > 0$.

The inequality in Theorem 3.2 is in fact an average comparative advantage criterion. This becomes clearer if we express the result in terms of the average products, $c_{i,j}$ of the linearized problem. Since $c_{i,j} = Q_i / l^*_{i,j}$, $c_{i,1} / c_{i,2} = l^*_{i,2} / l^*_{i,1}$. That is, with Q_i given, the country that can produce it with the smallest relative amount of labor has a comparative efficiency advantage in producing good i relative to the other good, k. Theorem 3.2 can be restated as:

Theorem 3.3: Average Comparative Advantage and L-Efficiency

The necessary and sufficient conditions for the $y_{i,j}$ to be L-efficient are: (1) $y_{i,j}$ uses all the labor of both countries and (2) the productivities satisfy $c_{i,1}/c_{i,2} \le c_{k,1}/c_{k,2}$ whenever i is produced in Country 2 and k is produced in County 1.

Efficiency in the Original Nonlinear Problem: We will now connect L-efficiency with efficiency in the original scale economics problem. To do this we attach to any feasible production plan P = {$l_{i,j}$} (i.e., to any set of input quantities that can produce the Q_i), its linearized plan P* defined by $y_{i,j} = f_{i,j}(l_{i,j})/Q_i$. This linearized plan, as we are about to show, satisfies the linearized problem. Note that the linearized *problem* depends only on the set of Q_i while the linearized *plan* P* depends also on the particular production plan P chosen.

Sufficient Conditions: We will first work toward a sufficient condition for efficiency in the original problem. This is done in what amounts to two steps. The first shows that if any output vector Q_i is efficient in the linear problem then it is efficient in the original problem with scale economies. Next, we invoke the result (Theorem 3.3) that in the linear problem comparative advantage plus full employment of labor in both countries are necessary and sufficient for efficiency. It then clearly follows that the comparative-advantage requirements suffice for efficiency in the original problem.

Lemma 3.4: Relation between Efficiency in the Original and the Linear Plans

If P is any feasible production plan then P* satisfies (3.1) and uses no more labor for any good. Equivalently: if the $l_{i,j}$ are any feasible production pattern, then $y_{i,j} = f_{i,j}(l_{i,j}) / Q_i$ satisfies (3.1) and $y_{i,j}l^*_{i,j} \le l_{i,j}$.

Proof: Consider any production pattern $l_{i,j}$, with $f_{i,1}(l_{i,j}) + f_{i,2}(l_{i,2}) = Q_i$, and satisfying (2.1). Now consider $y_{i,j} = f_{i,j}(l_{i,j}) / Q_i$. We will show that $y_{i,j}$ satisfies the linearized problem (3.1) and uses no more labor, i.e., $y_{i,j}l^*_{i,j} \le l_{i,j}$.

Clearly, $0 \le y_{i,j} \le 1$ and $y_{i,1} + y_{i,2} = (f_{i,1}(l_{i,1}) + f_{i,2}(l_{i,2})) / Q_i = Q_i / Q_i = 1$. It remains to show $y_{i,j}l^*_{i,j} \le l_{i,j}$. This will show both that $y_{i,j}$ uses no more labor and that it satisfies the inequalities of (3.1).

For $(i,j) \in D$, $e_{i,j} = f_{i,j}(l'_{i,j}) / l'_{i,j}$. Using economies of scale and remembering that in this case $l_{i,j}^* = l_{i,j}' \ge l_{i,j}$,

$$L(y_{i,j}\, l^*_{i,j}) = y_{i,j}e_{i,j}\, l^*_{i,j} = y_{i,j}\frac{f_{i,j}(l^*_{i,j})}{l^*_{i,j}}\, l^*_{i,j} = y_{i,j}Q_i = f_{i,j}(l_{i,j}) \le \frac{f_{i,j}(l'_{i,j})}{l'_{i,j}}\, l_{i,j}.$$

$$\text{eliminating the } \frac{f_{i,j}(l^*_{i,j})}{l^*_{i,j}} \text{ gives } y_{i,j}l^*_{i,j} \le l_{i,j}. \tag{3.3}$$

For $(i,j) \in D'$, $e_{i,j} = F_{i,j}(L_j) / L_j$, so

$$L(y_{i,j}\, l^*_{i,j}) = y_{i,j}e_{i,j}\, l^*_{i,j} = y_{i,j}\frac{F_{i,j}(L_j)}{L_j}\, l^*_{i,j} = y_{i,j}Q_i = f_{i,j}(l_{i,j}) \le \frac{F_{i,j}(L_j)}{L_j}\, l_{i,j}$$

$$\text{eliminating the } \frac{F_{i,j}(L_j)}{L_j} \text{ gives } y_{i,j}l^*_{i,j} \le l_{i,j}. \tag{3.4}$$

This ends the proof of the lemma.

We have as an immediate consequence:

Theorem 3.5

If a set of Q_i are *L*-efficient then they are efficient.

Proof: We will prove the equivalent statement: if the Q_i are *not* efficient in the original problem, they are *not* efficient in the linearized problem. Let us assume that the Q_i are *not* efficient in the original problem so there is a feasible production pattern P using *strictly less* than the total labor of the two countries. Then Lemma 3.4 asserts that P* is a solution to the linearized problem that uses no more labor, so it too uses less than the total labor of the two countries. This shows that the Q_i are not L-efficient. Q.E.D.

Combining this with Theorem 3.3 gives us a sufficiency condition:

Theorem 3.6

Sufficient conditions for the production plan P to be efficient are (1) the productivities $e_{i,1} / e_{i,2}$ satisfy $e_{i,1} / e_{i,2} \le e_{k,1} / e_{k,2}$ whenever i is produced in Country 2 and k is produced in County 1, and (2) the *linearized plan* P* uses all the labor of both countries.

The next theorem states that if the feasible production plan P *itself*, not P*, uses up the labor of the two counties, and satisfies average comparative advantage, it is efficient.

Theorem 3.7: Sufficiency of Average Competitive Advantage

Sufficient conditions for the specialized or extended-specialized production plan P to be efficient are (1) it has productivities $e_{i,1} / e_{i,2}$ that satisfy $e_{i,1} / e_{i,2} \le e_{k,1} / e_{k,2}$ whenever i is produced in Country 2 and k is produced in County 1, and (2) it uses all the labor of both countries.

Proof: If P is extended-specialized (which includes specialized) we have for goods with specialized production, $f_{i,j}(l_{i,j}) = (f_{i,j}(l'_{i,j}) / l'_{i,j})l_{i,j}$ because $l_{i,j} = l'_{i,j}$. For goods with shared production we have either $f_{i,j}(l_{i,j}) = (f_{i,j}(l'_{i,j}) / l'_{i,j})l_{i,j}$ if $(i,j) \in D$ or $f_{i,j}(l_{i,j}) = (F_{i,j}(L_j) / L_j)l_{i,j}$ because $l_{i,j}$ is already in the linear range. In either case we obtain $y_{i,j}l^*_{i,j} = l_{i,j}$ instead of $y_{i,j}l^*_{i,j} \le l_{i,j}$ in (3.2) or (3.4). Therefore, if P uses up all the labor so does P* and then Theorem 3.6 applies. Q.E.D.

Necessary Conditions: If we next consider necessary conditions there are certain things that are straightforward. A production plan P can not be efficient if an interchange of two industries between the two countries produces more of both goods, or equivalently, the same goods with less labor, so clearly n-good efficiency requires satisfaction of the two-good efficiency condition for every pair of goods as a necessary condition. However there is also a necessary condition related to the linear problem.

We start with this Lemma:

Lemma 3.8

Lemma: If the $y_{i,j}$ are integer (i.e., 0 or 1) and satisfy the labor-availability constraints (3.1), then, $P' = \{l_{i,j}\}$ where $l_{i,j} = y_{i,j}l^*_{i,j}$ is a feasible (specialized) production plan for the original problem.

Proof: Since the $y_{i,j}\, l^*_{i,j}$ satisfy (3.1) the $l_{i,j}$ clearly satisfy (2.1) – they use no more than the available labor. It only remains to show that $l_{i,j}$ can make the Q_i, i.e., that $f_{i,1}(y_{i,1}l^*_{i,1}) + f_{i,2}(y_{i,2}l^*_{i,2}) = Q_i$. For each i one of the two $y_{i,j}$ is 1, and the other is 0. Let us suppose $y_{i,1} = 1$ and $y_{i,2} = 0$. If $(i,1) \in D$, $l^*_{i,1} = l'_{i,1}$ so $f_{i,1}(l^*_{i,1}) = Q_i$ and $f_{i,2}(0) = 0$ so the $l_{i,j}$ make the Q_i. Clearly we have the same outcome if $y_{i,1} = 0$ and $y_{i,2} = 1$ provided that $(i,2) \in D$. We next consider the possibility $y_{i,1} = 1$ and $y_{i,2} = 0$ and $(i,1) \in D'$. As we remarked earlier when we were defining the linear problem, $(i,j) \in D'$ implies that $l^*_{i,j} > L_j$. This in turn implies, for $y_{i,1} = 1$, that $y_{i,1}l^*_{i,1} > L_1$. This means that the first inequality in (3.1) cannot be satisfied. That, however, contradicts the assumption that the $y_{i,j}\, l^*_{i,j}$ satisfy (3.1) so this case can not occur. The same reasoning applies to the remaining case $y_{i,1} = 0$ and $y_{i,2} = 1$ and $(i,2) \in D'$. Q.E.D.

This Lemma enables us to prove:

Theorem 3.9

If P is an *efficient* production plan, then the corresponding linear plan P* uses no more labor in (3.2) (or in (3.1)) than the minimizing *integer* solution to (3.2).

Proof: If we start with some efficient production plan, P, then by Lemma 3.4 its linearized plan, P*, satisfies the linearized problem and uses no more labor than P. If there were an integer y′ that solved the linearized problem with strictly less labor, it would give us by Lemma 3.8 a new feasible production plan P′ using no more labor than that, and therefore using less labor than the original P. This contradicts the assumed efficiency of the original P. Q.E.D.

If we start with a specialized $P = \{l_{i,j}\}$, it produces a P* with $y_{i,j} = f_{i,j}(l_{i,j}) / Q_i$. However we can verify that these $y_{i,j}l^*_{i,j}$ are exactly the $l_{i,j}$ of P itself. For example if $f_{i,1}(l_{i,1}) = Q_i$, $y_{i,1} = 1$ so $y_{i,1}l^*_{i,1} = l^*_{i,1} = l'_{i,1} = l_{i,1}$ This leads immediately to:

Theorem 3.10: A Necessary Condition for Efficiency

If a specialized P is efficient, its P* must be integer (perfectly specialized) and must be minimal among *integer* solutions to (3.2).

In other words P* solves the *integer* programming problem represented by (3.2).

Proof: This is a restatement of Theorem 3.9 for specialized P using the fact that, for specialized P, P* is integer.

This theorem tells us that for a specialized plan, P, to be efficient in the original problem, its linearized counterpart, P*, must be the minimizing solution of the integer (perfectly specialized) version of the (efficiency) programming problem (3.2).

Special Cases: We will next discuss some illuminating special cases. These cases illustrate the variety of outcomes that can occur when the classical efficiency concept is used in the scale economies model. In particular, they demonstrate that the proportion of efficient specialized equilibria will in some cases be very large, and in other cases very small.

Identical Production Functions: Assume $f_{i,1}(l) = f_{i,2}(l)$ for all i. Any equilibrium or any feasible production plan $P = \{l_{i,j}\}$ point provides a set of Q_i, and satisfies our condition (2.1). Let us suppose that P is specialized and that Country 1 is the producer in the ith industry. Then $l_{i,1} = l'_{i,1}$ and $l_{i,2} = 0$. Clearly, $(i,1) \in D$. If we also have $(i,2) \in D$, it takes the same amount of labor $l_{i,1} = l_{i,2}$ to produce Q_i in both countries because the production functions are identical, so $e_{i,1} = e_{i,2}$. If on the other hand, $(i,2) \in D'$ and Q_i cannot be made by the entire labor force of Country 2, it must be true that $l_{i,1} > L_2$. Then we have from economies of scale $f_{i,1}(l_{i,1}) / l_{i,1} \geq f_{i,2}(L_2) / L_2$ so $e_{i,1} \geq e_{i,2}$. In either case, the producing country, Country 1, has an $e_{i,1} / e_{i,2} \geq 1$ and the non-producer, in this case Country 2, an $e_{i,2} / e_{i,1} \leq 1$. This argument can, of course, be repeated if Country 2 is the producer and for all i. We conclude that the comparative advantage conditions of Theorem 3.7 are necessarily satisfied and that we have efficiency. So if $f_{i,1}(l) = f_{i,2}(l)$ we have efficiency for all specialized production plans or specialized equilibria.

To explain the next two examples we must allude briefly to some of the results of Gomory (1991, 1992) and Gomory and Baumol (1992). In these papers we showed not only that the n-good model has 2^n-2 specialized equilibria, but also that if we calculate for each equilibrium its relative national income $Z_1 = Y_1 / (Y_1 + Y_2)$ and its (Cobb-Douglas) utility U_1 for Country 1, and then plot the points (Z_1,U_1) in the Z_1-U_1 plane, the resulting 2^n-2 points lie in a well-defined region of the plane. This region has a characteristic shape and well defined upper and lower boundaries that can be computed. We also showed that the region between the upper and lower boundaries tends to fill up solidly with these equilibria as n increases.

A similar statement can be made about a Z_2-U_2 plane, if we choose to plot the equilibria with $Z_2 = Y_2 / (Y_1 + Y_2)$ as the horizontal axis and U_2, the Cobb-Douglas utility for Country 2 as the vertical axis. In *Figure 6*, (and later in *Figures 7, 8,* and 9), we combine the (Z_1,U_1) and (Z_2,U_2) plots. In *Figure 6* the dark dots are the points (Z_1,U_1) and the lighter dots the points (Z_2,U_2). The horizontal axis is Z_1 if read from left to right, and $Z_2 = 1\text{-}Z_1$ if read from right to left. *Figure 6* represents the equilibria for an 11-industry model. These equilibria give us a large set of specialized production patterns, whose efficiency has considerable natural interest.

With this background we can now discuss the next examples.

Productivity ratios that do not depend on the Q_i. Efficiency is also easier to understand when the $e_{i,j}$ depend on the size of the Q_i but the $e_{i,1}/e_{i,2}$ ratios do not. This occurs when, in each industry, one country has a consistent degree of advantage over the other over all Q_i or at least over a wide range of Q_i. An example of this is the case $f_{i,j}(l) = \varepsilon_{i,j}\, l^{\alpha i}$. In this case we find directly that the $e_{i,1} / e_{i,2}$ ratios for each i are given by the expression $(\varepsilon_{i,1} / \varepsilon_{i,2})^{\alpha i}$, which does not involve Q_i. These ratios are independent of the actual Q_i values and therefore are the same for all the different equilibria or specialized production plans. The ratios can be used to rank the industries in order of decreasing comparative advantage ratios, $e_{i,1} / e_{i,2}$. If we assign production of the first m industries in this ordering to Country 1 and the rest to Country 2, we obtain an equilibrium that certainly satisfies the comparative advantage condition of Theorem 3.7. We can create n-1 efficient equilibria in this way. These equilibria are shown in *Figure 7* for a 27-good model.

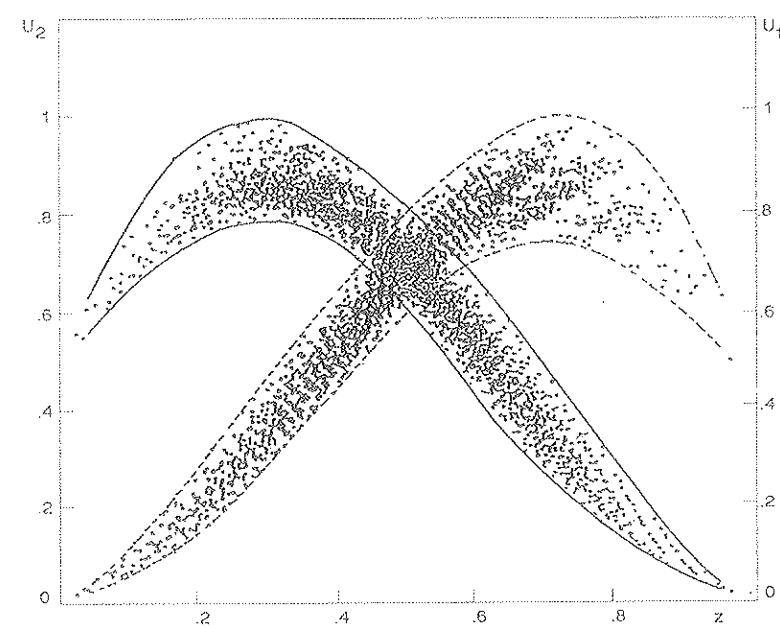
This example illustrates an important point. There are efficient equilibria that give very low utility for both countries, those that are near $Z_1 = 1$ or $Z_1 = 0$. At these equilibria one country makes almost everything and the other makes only one or two products. However, these products are made in such large quantities that the point is efficient, since there is no way to make more of those one or two while maintaining the quantities of the others. The disturbing implication

is that efficiency and welfare are almost completely divorced, except where Z_1 is held fixed, i.e., at a *given* level of relative national income.

Ratios that depend on the Q_i. One might think at this point that there is always some string of efficient equilibria to be obtained by finding the equilibria whose ordering corresponds to the comparative advantage ordering of the productivity ratios, $e_{i,1} / e_{i,2}$. However, what has simplified our work to this point is that the $e_{i,1} / e_{i,2}$ ratios with which we have dealt have been independent of Q_i, while in general the $e_{i,1} / e_{i,2}$ depend also on the size of the Q_i. This dependence can in fact exclude the possibility of efficient equilibrium points over wide ranges of relative national incomes. We can provide an example in which there are no efficient specialized equilibria in the equilibrium region of the Z_1-U_1 plane for $Z_1 > 0.5$. While this example is somewhat artificial it does show the sensitivity to the details of the classical efficiency concept in this setting. It therefore seems worthwhile to consider other concepts that may generalize the classical concept and improve its adaptation to a world of scale economies.

Figure 6

Regions of Equilibria from an 11-Industry Model



IV. LOCAL EFFICIENCY: THE EFFICIENCY COSTS OF NONSPECIALIZATION

We will call the $l_{i,j}$ *locally efficient* at Q if, roughly speaking, there is no nearby $l_{i,j}$, say, $l^{**}_{i,j}$, that provides more than the quantities Q_i. More precisely $l_{i,j}$ is locally efficient if there is some ε such that $|l^{**}_{i,j}-l_{i,j}| < \varepsilon$ implies that $f_{i,1}(l^{**}_{i,1}) + f_{i,2}(l^{**}_{i,2}) \leq Q_i$ for all i.

Local efficiency still rests on the idea that generates interest in the concept of efficiency. It still asks whether or not a better arrangement, i.e., one that generates larger quantities of goods than the Q_i, is possible. However, a *local* efficiency test only compares nearby arrangements, as common sense may suggest. Local efficiency makes behavior of our large numbers of equilibria much more coherent.

To see this we first need another concept – an *almost specialized production pattern* (aspp). In such a pattern, with the exception of at most one i, all production is specialized: $l_{i,1} > 0$ implies $l_{i,2} = 0$ and $l_{i,2} > 0$ implies $l_{i,1} = 0$. We will also assume as part of the definition that for positive $l_{i,j}$, $f_{i,j}(l_{i,j}) > 0$, i.e., that a positive labor input always yields a positive output at the point in question, since otherwise the point is automatically inefficient. Note that a perfectly specialized production pattern is always an aspp. With this definition we can state the main theorem.

Theorem 4.1

A sufficient condition for $l_{i,j}$ to be locally efficient is that the $l_{i,j}$ be an almost perfectly specialized production pattern.

This has the important Corollary: The production pattern of any perfectly specialized equilibrium point is locally efficient.

Proof of the theorem: The theorem is almost proved if we demonstrate the following lemma:

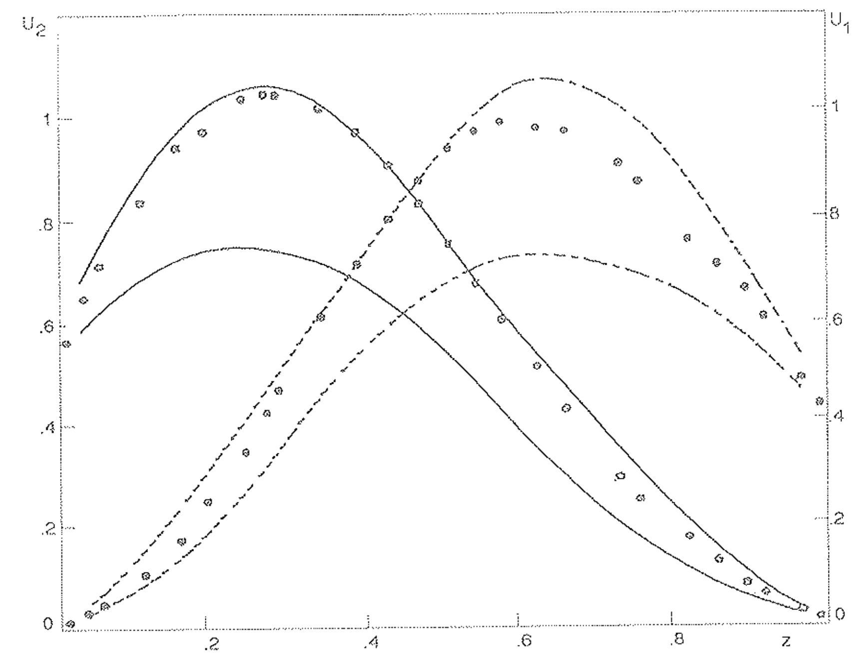
Lemma 4.2

If $l_{i,j}$ is an aspp, then for ε sufficiently small, any changes $\delta_{i,1}$ that strictly decrease the total labor quantity in Country 1, while maintaining the total output

of both countries at Q_i, will result in a strict increase in the labor used in Country 2.

Figure 7

Efficient Equilibria of a 27 Good-Model



That the lemma is plausible can be seen in the following way: if we attempt to decrease the labor $l_{i,1}$ on the one non-specialized good, we must increase the amount of labor $l_{i,2}$ in Country 2 to maintain the output of the ith good. If we then try to avoid an increase in total labor use in Country 2 we must decrease the $l_{j,2}$ in some other industry. However in this industry Country 2 is the sole producer, so any decrease in $l_{j,2}$ will result in a *very large* increase in $l_{i,1}$ because this industry is starting from scratch in Country 1. Now we no longer have a decrease in Country 1's total labor use. This reasoning can be extended to every possible situation as is shown in a proof available from the authors.

If we accept 4.2, then to prove the theorem we need only observe that if there is a $l'_{i,j}$ that makes at least Q_i, in every industry using the same labor supply, and makes strictly more of Q_j, then by contracting the production of the jth good until it is exactly Q_j we will underuse the labor of one of the two countries. If we suppose that it is Country 1, then $l_{i,j}$-$l'_{i,j}$ would give us a set of $\delta_{i,j}$ that underuses the labor of Country 1, and which makes the Q_i while not increasing the use of labor in Country 2. This contradicts the lemma and proves the theorem.

That the aspp condition of Theorem 4.1 is not too arbitrary can be seen from the following theorem:

Theorem 4.3

If the production functions $f_{i,j}$ have increasing derivatives (rising marginal products of labor), then *aspp is a necessary and sufficient condition* for local efficiency.

The proof of this Theorem is available from the authors.

Since efficiency implies local efficiency Theorem 4.3 has as an immediate corollary a result about classical efficiency:

Theorem 4.4

If the production functions $f_{i,j}$ have increasing derivatives there are *no efficient equilibria with more than one shared industry.*

This result helps to explain our emphasis on specialized and near specialized (aspp) production patterns[8]. In the case of increasing derivatives they are the only production patterns with even the possibility of being efficient.

Examples of production functions with increasing derivatives are all production functions of the form $e_{i,j}l^{\alpha i,j}$, with $\alpha_{i,j} \geq 1$, or any production function of that form preceded by an interval of zero output. An example of a reasonable production function *not* immediately meeting the criterion is a production function with a flat-bottomed average cost curve or one that starts out at zero, then increases sharply, and then becomes linear with a positive slope $e_{i,j}$ that is less than the derivative in the preceding steep portion. These are the functions of the type that underlies the extended-specialized production plans. However,

8. It is easy to show that in simple linear Ricardian model with two goods and two countries, varying only demand conditions, almost all efficient equilibria will be aspp, but *not* perfectly specialized, thus contradicting what the standard textbook examples suggest.

it can be shown that if the economy operates at a point in the linear range of such a production function, as it would be in an extended-specialized production plan, then we will generally have inefficiency with more than one shared industry. It can be shown that efficiency is not possible with more than one shared industry unless the $e_{i,1} / e_{i,2}$ ratios are the same in all the shared industries.

V. QUANTIFIED EFFICIENCY (λ-EFFICIENCY)

Although we see that all specialized production patterns are locally efficient, we are still interested in their efficiency in a more global sense. Now, however, we will not ask whether a given production pattern is efficient or not efficient but, rather, how bad or how good is it in terms of efficiency. We now seek a quantitative rather than a binary answer.

For this purpose we introduce as our quantitative measure, λ-efficiency. Consider any set of Q_i and its production pattern $l_{i,j}$. We will assume that the $l_{i,j}$ use up the total labor available in both countries. We will look for other production patterns $l_{i,j}$ that make the same set of Q_i, possibly using less of the total labor of the two countries, i.e., $\sum_{i,j} l_{i,j} < L_1+L_2$. We propose the

Definition: $\lambda = (\min \sum_{i,j} l_{i,j}) / (L_1+L_2)$ where the minimization extends over all production patterns that make the Q_i and do not require more than the available amount of labor in either country. We define λ to be the λ-efficiency of Q. Clearly $\lambda = 1$ coincides with the classical notion of efficiency, but now we can describe a point as having a λ-efficiency of, say, 0.77. This measures the percent of the total workforce required to make the original set of goods when they are made in the most efficient possible way. Thus, 1-λ is the proportion of the available labor wasted by the inefficiency of the equilibrium under study.

We will next discuss ways of finding the λ associated with a production pattern $P = \{l\}_{i,j}$. We will not be able to determine the magnitude of λ precisely, but will be able to obtain workable upper and lower bounds.

Underestimating λ: If v is the minimizing value of the linear programming problem (3.2) v is also an *underestimate* of the labor required to make the quantities Q_i using the $f_{i,j}$. This follows immediately from Lemma 3.4 which says that any feasible production plan P gives rise to a P* that satisfies the linearized plan and uses less labor. This gives us, by the definition of λ:

Theorem 5.1

$\lambda_u = v / (L_1+L_2)$ is an underestimate of λ.

Overestimating λ: Let v_I be the minimizing value of (3.2) when the minimization is not over all y but only over all integer y.

Theorem 5.2

$\lambda_o = v_I / (L_1+L_2)$ is an overestimate of λ.

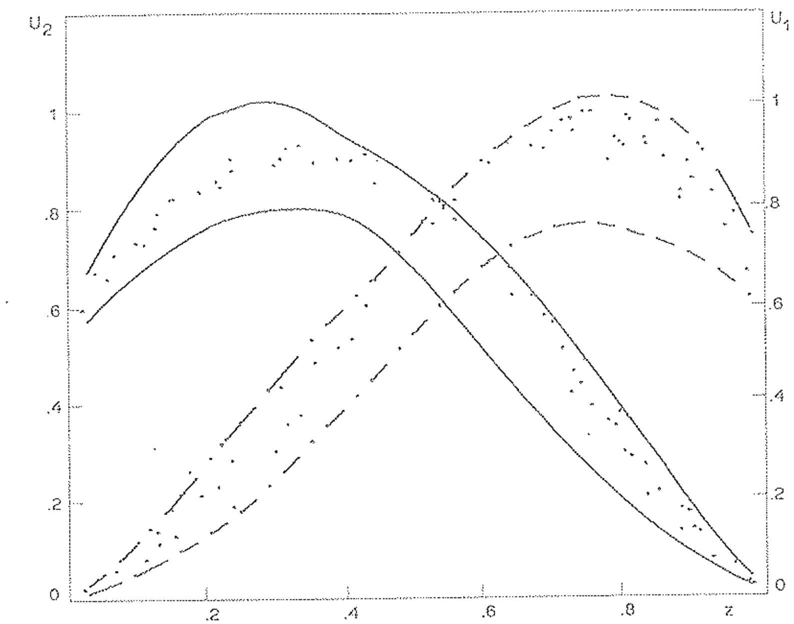
Proof: By Lemma 3.8 any integer solution of (3.2) is a feasible production plan. Any feasible production plan is an overestimate of the minimal amount of labor required. Therefore the minimum integer solution is a production plan and therefore an overestimate.

Simplifying the Calculations: Both the linear programming solution to (3.2) which gives λ_u and the integer programming solution to (3.2) which gives λ_o can be greatly simplified. The first problem reduces to the well-known knapsack problem and the integer problem can be reduced to a dynamic programming problem with one state variable. A description of the method can be supplied to interested readers.

Convergence of λ_u and λ_o for large problems: λ_o and λ_u will tend toward each other, and therefore toward λ for large problems in which no one or two industries use up a large proportion of the total labor force. Intuitively, this reflects the fact, mentioned in Section III, that the linear programming problem will have at most two non-integer components, and so its solution will not be far from being an integer solution itself. Therefore, if the two components don't matter much, the linear programming underestimate is not far from an integer programming overestimate. While this is plausible there is considerable work in getting these thoughts to be precise. A formal description is available from the authors.

Figure 8

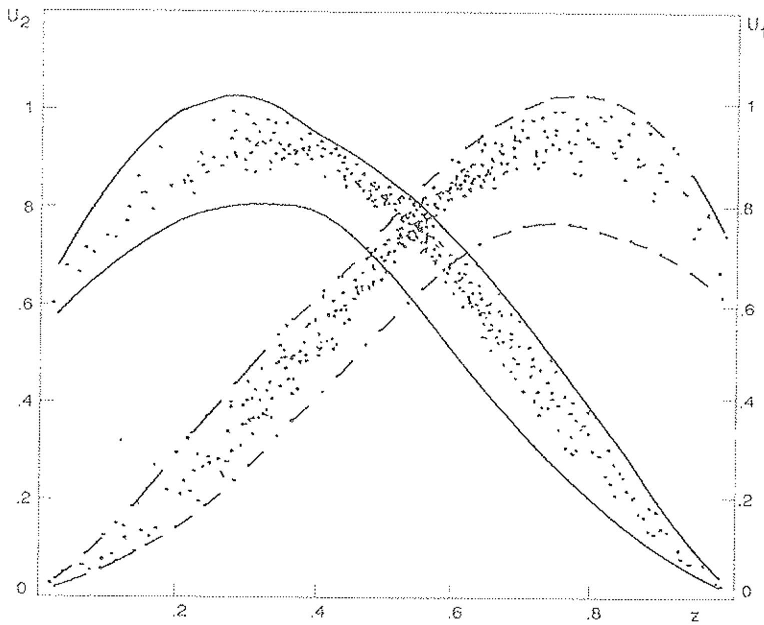
Equilibria with $\lambda_u > 0.98$



Efficiency in the Equilibria of a Region: We can now use the overestimates and underestimates to examine the efficiencies of the various equilibria populating the regions of equilibrium. In *Figure 8* we show the 98 equilibria in an 11-good model whose underestimate of λ, λ_u is > 0.98. In *Figure 9* we show the 449 equilibria whose overestimate of λ, λ_o is > 0.98. In these figures, and in other similar figures that we have examined, there is no particular tendency for the equilibria near the middle to be more efficient than the rest. There is, however, some tendency for the more efficient points to be near the upper boundary. This tendency is much more pronounced in the middle region than at either the right or left ends.

Figure 9

Equilibria with $\lambda_o > 0.98$



However, taking the region as a whole we may ask how efficient these equilibria are *on the average*. In *Figure 10* (a cumulative distribution) we have considered all the equilibria of the 11-good model. The height y of the upper curve gives the percent of the 2048 equilibria whose λ_u is $\leq$ x. The lower curve is a similar plot for λ_o. A plot of actual efficiency, if we could obtain it, would lie in between. The two vertical bars are the average λ_u on the left and the average λ_o on the right. The average efficiency lies in between, i.e., between 0.91 and 0.93. Considering the range of production efficiencies in the model parameters[9] the high average efficiency is a little surprising.

9. In the 11-country model the production function for the ith industry in the jth country was $e_{i,j}l^{\alpha_i}$. The α_i were between 1 and 2. The 11 values for the $e_{i,1}$ were (1.00, 1.02, 0.70, 0.94, 1.24, 0.60, 0.70, 0.77, 0.50, 1.10, 0.90) and the 11 values for the $e_{i,2}$ were (0.52, 0.71, 0.91, 0.92, 1.01, 1.23, 1.30, 1.02, 0.30, 1.20, 0.70). The 27-country model was similar.

Figure 10

λ_u and λ_o for an 11-Good-Model

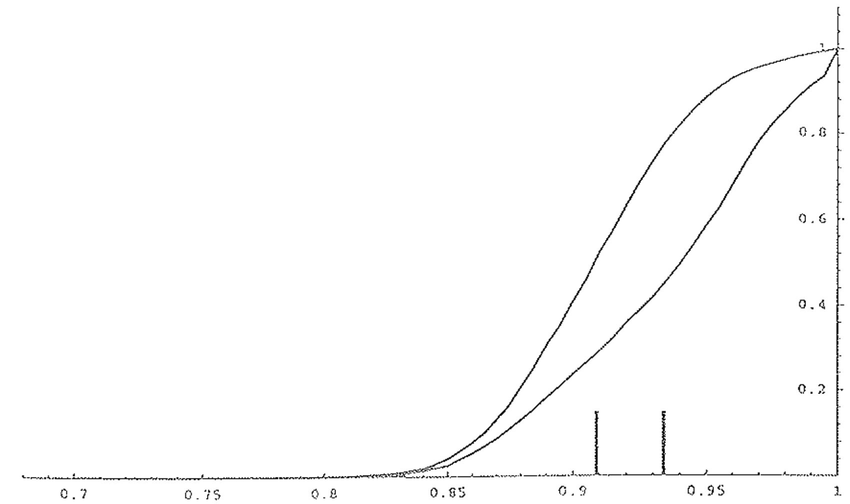
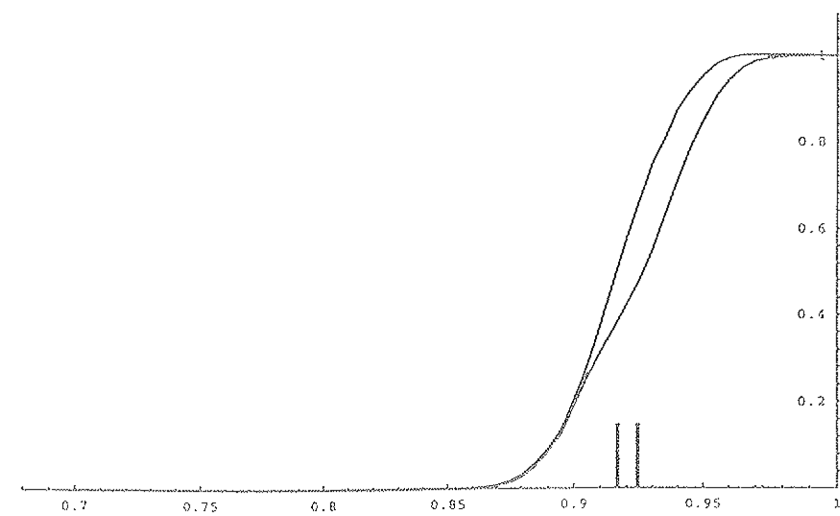


Figure 11

λ_u and λ_o for a Sample from a 27-Good Model



In *Figure 11* we have the same plot as that in *Figure 10* but this time we have taken a random sample of 2,000 points from a larger (27-good) model. The convergence of the two curves and the improved estimates of the average efficiency are exactly what one expects from the convergence discussed above.

VI. CONCLUSION

As promised in the introduction, we have demonstrated that there is a wide range of states of efficiency, in the classical sense, that are possible for the many equilibria that exist under economies of scale and for similar production relationships. We have shown the profound differences in efficiency theory from that in the classical case of diminishing or constant returns, proving that equilibria need not be efficient and that even efficient equilibria need not satisfy comparative advantage. We have discussed examples in which all 2^n-2 specialized equilibria are efficient, and examples in which there are very few efficient equilibria. We have shown how the concept of comparative advantage can be modified to adapt it to the n-good case and especially to the specialized or extended-specialized solutions that are important in a world of scale economies, demonstrating that comparative advantage is sufficient but not necessary for efficiency. We have introduced two new efficiency measures, local efficiency and λ-efficiency that seem to function more uniformly than the standard classical concept in the economies of scale setting, while retaining many of the properties that provide interest to the notion of efficiency.

The subject is clearly not of academic interest alone. It relates immediately to the role of laissez faire and government intervention in international trade. The classical analysis showed good reasons to believe that in a world of diminishing returns unimpeded market forces can be relied upon to do a reasonably good job in promoting efficiency in the trade process. The current paper, along with a number of recent writings dealing with the role of scale economies in international trade show that here matters are less simple. It does not follow that mindless government intervention, much less unfettered protectionism is the way to go. But the analysis shows that economic welfare may be enhanced in some circumstances if the governmental takes on some role. Exploration of the appropriate role is only beginning and this paper is intended as a contribution to the process.

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SUMMARY

In the presence of scale economies a country that happens to be the exclusive producer of a commodity will be able to retain its monopoly against efforts of others to enter on a small scale even if that firm has neither absolute nor comparative advantage in its production. Hence equilibria can violate comparative advantage, be inefficient and yet be stable. Moreover, equilibria that violate comparative advantage can be efficient. In the article, sufficient efficiency conditions for the scale economies case are provided, and, two new concepts, local efficiency and (quantitative) degree of inefficiency are explained. The analysis confirms that in a world of scale economies market forces cannot be relied upon always to yield an efficient equilibrium.

ZUSAMMENFASSUNG

Ein Land, welches der exklusive Hersteller eines Gutes ist, wird beim Vorliegen von Skalenerträgen seine Monopolstellung gegenüber Konkurrenz von kleiner Grösse selbst dann halten können, wenn es weder absolute noch komparative Vorteile in der Produktion besitzt. Daher gibt es Gleichgewichte, die das Konzept der komparativen Vorteile verletzen, ineffizient aber trotzdem stabil sind.

Allerdings können solche Gleichgewichte auch effizient sein. In diesem Aufsatz werden hinreichende Effizienzbedingungen für Skalenerträge aufgezeigt und zwei neue Konzepte, lokale Effizienz und ein (quantitativer) Grad an Ineffizienz erklärt. Die Analyse bestätigt, dass beim Vorliegen von Skalenerträgen die Marktkräfte nicht immer zu einem effizienten Gleichgewicht führen.

RÉSUMÉ

Au cas d'économies de l'échelle, un pays qui est le seul producteur d'un bien peut être capable de préserver son monopole malgré les efforts d'autres pays d'entrer dans la production à une échelle plus petite, même si le pays tenant le monopole n'a ni un avantage absolu ni relatif dans la prodution. Il en suit que ce genre d'équilibre peut contredire le principe de l'avantage comparatif, être non-efficient et être stable. De plus, des équilibres qui violent le principe de l'avantage comparatif peuvent être efficients. Dans l'article les conditions suffisantes pour le cas d'économies de l'échelle sont données, et deux conceptions nouvelles, celle de l'efficence locale et celle du degré (quantitatif) de l'inefficience sont expliquées. L'analyse confirme que dans un monde avec des économies de l'échelle les forces du marché ne produisent pas toujours des equilibres efficients.