EDUCATION IN APPLIED MATHEMATICS

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VI. POST-DOCTORAL EXPERIENCE

Introduction by G. E. Forsythe. The topic of this Conference, as you know, is Education in Applied Mathematics, and I am very much impressed with the fact that education does not stop with a Ph.D. degree. I at least did not even start the present field that I am working in until seven years after I got a Ph.D., and nothing that I teach even to freshmen was particularly thought of when I took a Ph.D. The language I teach—I am a computer scientist not a mathematician—the language I teach was not invented then. So that in computer science, at least, we are less able even with freshmen to fall back on those things that we learned in our own education.

Now, while the faculty at major universities and major industrial laboratories like those represented here are completely used to continual education throughout their lives, I think it is safe to say too many Ph.D.'s in mathematics rather coast through their career pretty much on what they learned in graduate school.

Well, we are here to consider, then, post-graduate education, and perhaps to decide whether this is a lifelong process or not. The first speaker is Ralph Gomory.

R. E. GOMORY

I have a quotation here which I think indicates that education can start very late in life and also that people can become interested in applied mathematics very late in life and for a variety of reasons. I am going to read this quotation which was written by quite an old man.

"Had it not been for the guidance of the thought of Mao Tse-tung I daresay that I would surely have been contented with what I was. With a pot of tea and a cigar I would occupy myself in my study of mathematical problems which were so dear to me and with which I was familiar, oblivious to the buffeting of the four seas and the storms on the five continents. Without exerting undue effort I would write so many learned treatises a year so that my name might be known and I might live in peace and comfort. I would teach a little, give guidance to students, and handle my specialized knowledge without reservation and without discrimination to the younger generation. In this way it appears I would have done my duty. But would I really? No."

Then it goes on at some length to explain the process of enlightenment which overcame this man.

Finally, he says:

"The habit of practicing what one has learned and applying what one has practiced and of combining learning and application is vastly different from the way of study of going around in circles among concepts and indulging in empty discussion of terms with which I was familiar. This enables me to realize profoundly that class struggle, production struggle, and scientific experiment are united in an integral whole, that there cannot be a new type scientific worker who can carry out scientific experiments well but whose idea of class is indifferent and whose production practice is vague. There might have been such scientists in the past but there are no more. They could not be compared with the scientists who have grown up with the help of Mao Tse-tung and who combine in themselves 'redness' and expertness."

One thing I realize is how different our attitudes are and also what strange and different reasons, good and bad, we all have for reaching them.

Here apparently is a society, the current Chinese one, in which a certain type of applied mathematics is considered quite fashionable if not mandatory.

Having illustrated in this exaggerated way that change in attitude towards mathematics is still possible at an advanced age, I am now ready to talk about education in Applied Mathematics *after* the doctoral level. Actually, you can divide that education in two parts: education that perhaps could be derived from industry and government and that which one would acquire in a university. I am not going to say anything about the second because I do not know anything about it, not being in that very much. I will leave that to all the other discussants. So I am really going to talk about the industrial part, and in fact, as you will see, I am not even going to cover very much of that.

First, I would like to ask what happens to a person after they get a Ph.D. Do they go to industry, do they go into the universities? In which environment are they, anyway? I have some figures here. This is for 1963.

Academic institutions attracted by far the largest proportion of Ph.D.'s in mathematics—by the way, I will be talking about Ph.D.'s—72 percent. Industry, with its comparatively higher salaries, attracted 17 percent of new mathematics doctorates, 5 percent went to research institutions, and 6 percent were employed by the government.

The next year the picture was even more marked, that is, that academic life again attracted the largest proportion of new Ph.D.'s in mathematics, 83 percent going into academic life. Industry attracted the next largest number, 8 percent, and research institutions and government employment attracted 3.7 and 4.1 percent.

Let me ask briefly what happens to this small group, and one should realize now that it is a very small percentage of people that go into industry. They can be employed in a variety of way. They can be in research and engineering units, operations research units, mathematics units, statistical units, computing laboratories, and something called the general technical staff, which is just a catch-all.

There was a survey of what mathematicians do in industry, and I am just going to select one or two facts, which I think are pertinent to our discussion, from this survey.

Now, the survey, I want to point out, is not a survey of Ph.D.'s alone but of all mathematicians in industry and government. However, it is possible to separate out the Ph.D. part.

One salient fact is that something like fifty to fifty-five percent of the mathematicians do applied or basic research in engineering and the natural sciences. About 15 to 20 percent do nontechnological research or services allied to marketing, production, sales, promotion, or distribution, and the rest do a miscellany of things, administration, teaching, and so forth.

So the general picture is that, what one might refer to as the classical fields dominate, with more than half the people working on what you might have called the classical areas of applied mathematics, but that the new fields, that is perhaps too sweeping a way of describing it, are a substantial nonnegligible fraction, and this survey is already now about five years old.

So I think that those people who tend to regard mathematics, applied mathematics, as consisting entirely of the applications to science and engineering have a substantial measure of truth in that statement; it covers over half of the outside activity, but there certainly is a nonnegligible other part.

So much for a few facts about the general background. From now on, I am going to really confine myself to the mathematics units in this list of various units that people can belong to. I think the things I am going to talk about apply also to other positions but less well, I think probably to computing laboratories which do research and to operations research units to some extent.

I am going to make a very, at least a moderately, specific suggestion, and really I am just going to go on about this point from now on, which is that I think that we should try to get the industrial mathematicians and government people to play a real role in the educational process. It is a role which has not been played very much up till now.

For example, what I want to suggest is that a period spent in industry or government should be considered a normal part of the life cycle of an applied mathematician—of some of them, of course, but not all.

Why do I make a suggestion like this to an audience like this, which I think sometimes has even to be reminded that there is anything outside of universities. It is because I really think that there are real benefits, both to the individuals and to the subject matter of mathematics which can be obtained in this way. To make this seem plausible, I will discuss it in a very general way, then I am going to try to be more specific. Certainly it appears that applied mathematics, however we define it, seems to involve both mathematics and some degree of contact with some element of the world outside of mathematics. Much science and some engineering are directly represented on the campus. However, many aspects of engineering and the whole bundle of problems involved in production, distribution, and so forth, are not well represented.

Therefore, if you want to get in contact with these things and you want to have a chance to mathematize them, or if we want them to be introduced into mathematics in the way of new topics, it would seem that some direct exposure to them would be very valuable, and I stress the advantages of really coming in contact with the problem.

Now, I guess you can sort of see in the abstract that it is plausible that this might be a good thing. However, when I think about why I am convinced that it is a good thing, I find that it is not really because of these plausibilities at all, but because of my own experience. I have seen this sort of thing work for many people. I think Cohen and Toupin can comment on the influence that an environment full of topics has on mathematical development. But I guess really my conviction comes from the effect it has on me. So I am going to give as an example some of my own activities which are supposed to illustrate the interaction of a new class of problems, one which is not well represented on the campus and in mathematics.

This is supposed to represent this interaction (at the blackboard). There is sort of an underlying set of problems here, which are, perhaps one could say, the desire to obtain some understanding about some of the finite combinatorial problems. The industrial aspect really provides stimulus and special cases, and it provides something else which I like and which I think perhaps many people do, which is just the joy of actually doing something with real worth, because part of the enjoyment of this whole thing is that; you know, there is the mathematical side and there is also the other side.

Let me try to explain how these things work and why I think this is beneficial, and why, to some extent, these are things that cannot be obtained in any other way.

The way to decipher this thing is that the boxes which have thick lines around them represent the outside world, definite applications, and the boxes which have thin lines around them are, let us say, mathematical concepts, and I am going to illustrate how the two things interact.

Now, the work I am describing is not mine—it is mine and other people's at the same time. One can start off, as I did, with an interest in combinatorial problems and using the techniques of integer programming and decomposition methods. It does not even matter too much if you do not know what these words mean. Let us say, however, basically we are dealing with systems of linear inequalities and when the word "integerprogramming" appears, it means that we are dealing with Diophantine type problems involving systems.

Putting these two notions together one sees that there are many very large systems of linear inequalities, provided they have a special structure. And then when this happens you start to look around and all of a sudden the whole world around you seems to be teeming with examples. Everything you look at suddenly turns into a very huge matrix with thousands of columns.

In this state I first heard about the problem of the steel beams. Well, it turned out that when you make a bridge you make steel beams out of long pieces of steel, and the problem of cutting up the steel so as not to waste any is a very difficult, painful one.

So I went down to Philadelphia with Paul Gilmore and looked into this question. Well, the more I studied the steel beam problem, the more I realized we could do absolutely nothing with it because there were very few beams alike in a bridge. Each beam was a different length and our techniques simply could not cope with that situation. So I returned back to my theoretical studies.

Again I heard of a problem. This time it was the problem of cutting up enormous rolls of paper. Rolls of paper are very long. If you see them, they stand this high from the floor and they are about ten feet long. The problem is that paper is produced in these large quantities. There was no use at all for such a large roll and they had to cut it up. And, again, in cutting it up into the various widths the customers demand, pieces are left over and they are wasted, and this is a source of great anguish.

Well, investigating this, it turned out that our techniques actually worked, and we went to the paper mills and there we encountered some of the problems which you have heard discussed, just as Montroll talked about training students to be patient with the physicists because their explanations are often unclear. We encountered this problem. We talked to the people who set the knives which cut the paper and tried to find out how they do it. They represent science to us and their art of setting the knives plays the role of physics; their explanations play the role which you are accustomed to having from the physicists, and I am sure they suffer from all the same difficulties that physicists do, sometimes compounded by the fact that they do not necessarily speak English. So we encountered in exaggerated form the same difficulties.

Now, in fact, we were able to do quite a bit with this problem and that is why it has a "U" next to it on the blackboard. "U" means things that are in use, because I try to distinguish between applications which one finds in many journals, which are things one hopes some day might be used and which may or may not, and things which actually are used.

The necessity of making this work in a reasonable length of time forced us to consider very carefully our calculations. Let me remind you of the underlying theme here, though, which is at all times combinatorial and Diophantine systems of equations.

One of the reasons why I pursued this particular thing is, it had a very interesting mathematical structure that was suited to my interest, which was the key subproblem to be solved in order to cut up the rolls of paper. It was a subproblem within a larger one. We felt at the time that this was a good thing—because anything that would be true in general for the large system certainly would have to be true for a system consisting of one inequality and perhaps we would see it. For many years we did not see it, but then finally we did.

Let me return to the application side. As we pursued this, we started to see many other things, namely, that the problem of paper trim was a manifestation of something larger, which was basically economy of scale, that is, it is cheap to make things in large clumps. Paper is made big, glass is made in enormous long ribbons. Wood is made in trees, not in tiny little pieces. So in order to use all of these things we have to cut them up into a variety of pieces to be used. Once you start thinking along these lines you realize the world we are in is full of this type of thing, even transportation problems basically, freight especially comes in boxcar size for reasons of economy. But when you ship boxes you cut up the space inside into little pieces. So that we realized that there was a large class of problems, only some of which we could cope with. In fact, we have a huge bundle of related unsolved problems.

You see, really, the only sort of thing we have learned to do up to now is cases in which rectangular objects, like large plates of glass, are cut by the cutting process into rectangles. This process can be repeated but you wind up with something quite intricate. A rectangle has a very nice property, which is that when you cut it, it again is really a good rectangle. It is also true that triangles have a nice property, namely, if you cut them, you produce triangles. But we do not understand much beyond that. Yet it seems to be sort of a key property.

Well, let me continue a little bit with the application problem. We have developed a method for cutting up wood, even taking account of the knots, which had to be left out. The furniture company put it in—you know, they take the wood and cut it up to make furniture—and for about a week it buzzed away beautifully, saving some 10 or 15 percent of the wood. At that point the air was full of dust from the wood. Someone had to stand over the thing and put something over the knots so that the location of the knots would be fed into the computer. The mechanism kept jamming and getting gummed up.

This application does not have a "U" next to it. It never recovered from this disastrous effect.

On the other side is the glass problem, which is a problem we could only solve a little bit because it has extra complications; people took it, the little bit we could do about it, and made their own modifications. I do not even know what they are doing any more, but they used it.

Finally, let me say that the "corrugator" was a very serious problem because we again went into that with the idea of using our methods of cutting things up and we got a very educational surprise, which was, that that was not the problem there at all. It is pretty easy to explain how to cut up long cascades of corrugated paper because the cutting methods are sort of restricted, the possibilities are not great.

So we dashed in and we thought, "Well, how nice." But it turned out at the end, when they took our solutions they just fell apart. They fell apart because of a very simple reason. The way they cut the corrugated paper is something called the triplex, which is three axles with circular knives on them; the corrugated paper streams along under the knife and it gets cut. While it is streaming along here a man is adjusting the knives on one of these back axles so that they can cut different combinations of sizes; and after it has run a little while, they produce enough of that mixture of sizes, they rotate this whole thing and then prepare the next one. They hate to have it stop, that is the whole idea, they do not want it to stop.

Well, our solution gave such short runs, forgetting the setting of the knobs, that the man could not set them up.

We brooded about this for a long time because it was obviously a common part of the problem, although at that time we could not see any relation. We eliminated one of the axles. Now we were going to solve the wrong problem again. It turned out at that point that this converted it into a traveling salesman's problem. Do not worry about what the traveling salesman's problem is, although I think there are people here who know. It is a recognized problem for which no reasonable method of solution, such as finding an answer, is known.

But it turned out that we had a special case if one thought about it long enough, and we were able to devise quite a good method for that. Then in thinking about that we realized we had really solved something much more general. What we had was the concept of what we called the one-stage variable machine, which enabled us to solve all sorts of sequencing problems, in what order, to do things with a set-up time demand, roughly speaking, like a long run, then a short run.

We had turned up the notion of the one-stage variable machine. It turns out, whenever you have a sequencing problem on a machine, if the state can be characterized by a single variable, then you can solve the problem.

For instance, you know the main thing about a furnace is how hot it is. If you are content with merely knowing its temperature and if the only things that matter are things affecting temperature, then it is possible to solve the sequence.

Another problem of this type is the tape on a computer. All you have to know is where you are on it. If you want in succession to pick up batches of information on the tape at intervals and then move to some other batch, then to some other batch, this is a sequencing problem on a one-state variable machine. It is solvable. But do not think it is easily solvable. There is no sort of an intuitive way to do it.

Let me return, then, to the main theme which goes through here. We were working away on the one-dimensional problem of integer programming, if you will, which was the key tough problem. We devised special methods for this. In studying the special methods we discovered a certain peculiar periodicity which led us to a periodic theory of one-dimensional integer programs. This we were finally able to generalize, and from this emerged the asyomptotic theory of integer programming, which connects it with ordinary linear programming and enables you to convert from one thing to the other.

This asymptotic theory depended in turn on maximizing a certain form of a finite group, which introduced a whole bundle of problems of this sort.

None of this would have occurred without these interactions. We would never have come to these questions without the traveling salesman problem, which is leading us now into various special cases of this very difficult and impractical problem.

I will not go any further; but I will say there are other branches to this interaction and they all have this characteristic, that is, that one starts off with a certain set of interests, and by bouncing off the problems and doing the special cases, all sorts of very remarkable things can come out, even to such topics as points of topology with which we are now involved.

I think that conveys the idea which I wanted.

Now, the moral of this is, at least the thing I am trying to convey to you is, that there really is something to be gained by this kind of direct exposure and direct experience with these problems. Now, if you are willing to accept that or you think about it, we should then turn to the question of how can we realize it, how can we get this kind of experience into people's situations?

Well, I think it can be done, and I think all I have to do is envisage the industrial group not as a dead end, but as a place which people go through, rather as people go for training to the university and then go on to some other place.

Let us see if it is not possible to use the independent industrial groups in this way. People could go there for several years, or whatever period of time is appropriate, get exposure to these topics (I am trying very hard to stay away from the word "problem," since I do not envisage us as problem solvers but rather as developers of areas) and then return to the universities, bringing with them new fields and new topics, and thus adding something new and lively to the subject matter of mathematics.

Is this possible? Is it a feasible thing to do? I think it is. I would say the main obstacles are not even sort of physical or institutional ones, but they are such things as suspicion and prejudice. Suspicion, for example, which might be embodied in the notion which I have encountered when I have mentioned these things to people, that everything I have said so far is just a device to get people to come to our laboratory. Let me assure you it is not, if for no other reason than the fact we just do not need people at all. We have many more applicants than we can possibly do anything with, and very good ones. Or prejudice, as embodied in such statements as "Only fourth-rate people go into industry." Let me again say no, it just does not fit the facts; and let me say that as far as our own group, one which I really know well, is concerned, there is practically no one in it who just within the last calendar year has not gotten an offer of a university position. I include such things as professorships at Berkeley, and so forth.

I think that some of these notions just are not correct, and I think that if we forget them we can get on with making something along these lines.

I could, of course, describe various specific schemes: forms of exchange such as to invite someone from an industrial unit to spend three years in teaching on the campus, and for universities to send some of their people for three years into industry, people coming right after the Ph.D. You can have people in industry helping with the Ph.D. programs. However, I will not proliferate these schemes, because I really think that if once you adopt a notion that there is something valuable to be gained by this experience, and that you are dealing with people who are pretty good and who are really interested in helping, and this is not a trick, such schemes can be worked out, and that the exact details will be different in each case.

There is one thing which I think would be quite helpful, that would be to improve the nature of the life of a mathematician in industry. I think if we made it a little more the way it is in universities, it would make this flow back and forth easier for people: if we had something like tenure, if we had not only in practice, but on paper, independence from our superiors. This is mostly a question of what one does, but it is not on paper, and therefore people not familiar with the structure are very reluctant to enter into it.

I think there is room for a great many improvements in the way that industry deals with the scientist that would make this flow back and forth easier. I think that these things are coming, but at any rate, they are something we have not done anything about as yet.

Basically, I would say, that if you realize that there is something to be gained by this exposure, if you want to have it, the forms can be worked out, and I think we can provide a new way of life, and an exciting one, for a certain number of people involved in applied mathematics.

DISCUSSION OF SESSION VI

B. H. Colvin. I think I might best contribute to this discussion by trying to supplement Dr. Gomory's remarks with particular reference to our experience at Boeing.

Let me emphasize first of all that I am going to talk about research mathematicians in a corporate research laboratory. This represents a rather special environment which provides unusually favorable opportunities for worthwhile mathematical activity. The remarks which I made earlier concerning the overall critical situation in training mathematicians for nonacademic work would certainly not apply to this group. This is a small group of about 20 people with Ph.D. training. I think you would all agree that they are well trained and, in general, quite experienced people.

I address myself to the question: "What experience do these people get in industry which contributes to their professional growth?" First, let me briefly list ten or a dozen forms of activity which do contribute in one way or another to the postdoctoral training of our staff, and then make a few remarks which are suggested by Dr. Gomory's comments.

Generally speaking, the staff at the Mathematics Research Laboratory is concerned with research, consulting and collateral activities in mathematics. It is convenient for me to group these activities into those which are internal to the company and those which are primarily external.

INTERNAL ACTIVITIES. *Research*. With regard to research activity, I should say that we choose our staff very carefully with regard to their training, interests, attitudes, and general areas of preference in research. Consequently there is no problem concerning what they may wish to work on. We know in advance that it will be an area of mutual interest to the man and to the company.

Consulting. In the area of consulting we do not have any specifically prescribed consulting duties. However, we recognize consulting and advisory duties as a rather important part of our responsibilities to the company and we spend a reasonable fraction of time on such consulting activities. By consulting activities I mean the rather broad consulting activities which Dr. Gomory has so admirably described, in which one seeks to identify and to formulate worthwhile research areas—research problem areas on which one can reasonable hope to make some effective contribution from the point of view of applied mathematics.

Since I have inadvertently used the term "applied mathematics," let me just say that we do not ordinarily use this term. Our organization is called the Mathematics Research Laboratory.

The professional development of our staff is influenced to a major extent by their experience in consulting and research. These kinds of activity are well understood by all of you and I shall not elaborate upon them.

Intramural teaching. A third activity which contributes to the development of